TASK 3

Name____Date__________________

Scuba Math

Part 1:

Serena and Dion took a scuba diving course while on vacation in Hawaii. On their first scuba diving trip, Serena swam toward the surface as Dion began his dive. The tables below represent their depth in feet with respect to time in seconds.

Time (seconds)	Serena's Depth (feet)
0	-90
10	-80
20	-70
30	-60
40	-50

Time (seconds)	Dion's Depth (feet)
0	0
20	-20
40	-40
60	-60
80	-80

1. Assuming they continued swimming at the same rate, after how many seconds will Dion and Serena be at the same depth? Show all work and use equations to support your answer.

Part 2:

TASK 3

On the second day of their vacation, Serena went scuba diving again. Later, Dion also went scuba diving. Once Dion began his dive, their depth in feet, y, can be described by the following equations, where x = time in seconds:

$$\text{Serena: } y = -70 + 2x$$

$$\text{Dion: } y = -1.5x$$

The following algebraic strategy can be used to determine the time at which they are at the same depth.

Step 1: $-70 + 2x = -1.5x$

Step 2:
$$\begin{array}{rcl} -70 + 2x & = & -1.5x \\ +1.5x & & +1.5x \\ \hline -70 + 3.5x & = & 0 \end{array}$$

Step 3:
$$\begin{array}{rcl} -70 + 3.5x & = & 0 \\ +70 \qquad & & +70 \\ \hline 3.5x & = & 70 \end{array}$$

Step 4: $\dfrac{3.5x}{3.5} = \dfrac{70}{3.5}$

Step 5: $x = 20$

2. Next to the steps listed above, describe each step in the solution strategy and explain why each step makes sense algebraically.

3. Verify that $x = 20$ represents the x-value of a solution to the system of linear equations. Show all work and explain your reasoning.

LESSON GUIDE 3

Scuba Math

Why this lesson now?

In the previous tasks, students made sense of the solution to a system and then solved a system given a context. In this developing understanding task, students solve and make sense of a system represented by a table of values and a context. Since the exact solution cannot efficiently be determined using a graph or guess-and-check, students are pressed to determine the solution algebraically. They are then pressed to analyze an algebraic solution, making sense of how the properties of equality are applied.

Task 3: Scuba Math

Part 1:

Serena and Dion took a scuba diving course while on vacation in Hawaii. On their first scuba diving trip,...

See student task sheet for the complete task.

Mathematical Content Standards	8.EE.C.8.A	Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
	8.EE.C.8.B	Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*
Mathematical Practice Standards	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP4 Model with mathematics. MP5 Use appropriate tools strategically. MP6 Attend to precision. MP7 Look for and make use of structure.	

Essential Understandings (EUs) **Greyed-out portions not addressed in task or lesson.*	• The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs (x, y) that make both equations true statements or satisfy the equations simultaneously. • The solutions to a linear equation in two variables can be represented graphically by a line consisting of all of the points represented by the ordered pairs (x, y) that make the equation a true statement or satisfy the equation. • The solution to a system of two linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because the intersection point(s) make(s) both of the equations true statements or satisfies both of the equations in the system simultaneously. • The properties of equality, including the reflexive, symmetric, and transitive properties of equality, along with substitution form the basis of algebraic methods (substitution and the addition/elimination) for finding the solution to a system of equations, or recognition that the system has no or an infinite number of solutions.
Materials Needed	• Student reproducible task sheet • Rulers, graph paper, calculators (optional)

SET-UP PHASE

I'd like a student to read the task aloud while everybody else reads along silently. Without giving anything away about the solution, who can explain what this problem is all about? Has anybody ever been scuba diving? This picture shows the equipment that is needed. Why do you think a wetsuit is worn? Does anybody know how long divers can stay underwater? Is there a limit to how deep they can go? I'm going to give you 5-10 minutes to work independently on Part 1 before sharing ideas with your group. You can move on to Part 2, but we will also spend some time on Part 2 after we come back together as a group to discuss Part 1. Graph paper, calculators, and rulers are available at your tables to use as needed.

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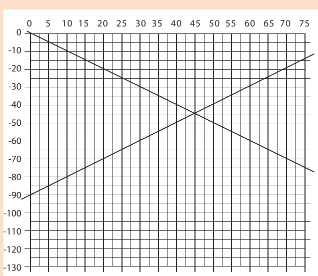
EXPLORE PHASE

<table>
<tr><th>Possible Student Pathways</th><th>Assessing Questions</th><th>Advancing Questions</th></tr>
<tr><td>Extends the tables of values.
<table>
<tr><th>Time</th><th>Serena's Depth(ft)</th></tr>
<tr><td>0</td><td>-90</td></tr>
<tr><td>10</td><td>-80</td></tr>
<tr><td>20</td><td>-70</td></tr>
<tr><td>30</td><td>-60</td></tr>
<tr><td>40</td><td>-50</td></tr>
<tr><td>50</td><td>-40</td></tr>
<tr><td>60</td><td>-30</td></tr>
<tr><td>70</td><td>-20</td></tr>
<tr><td>80</td><td>-10</td></tr>
</table>
<table>
<tr><th>Time</th><th>Dion's Depth(ft)</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>10</td><td>-10</td></tr>
<tr><td>20</td><td>-20</td></tr>
<tr><td>30</td><td>-30</td></tr>
<tr><td>40</td><td>-40</td></tr>
<tr><td>50</td><td>-50</td></tr>
<tr><td>60</td><td>-60</td></tr>
<tr><td>70</td><td>-70</td></tr>
<tr><td>80</td><td>-80</td></tr>
</table></td>
<td>Tell me about the values in your table. How can you determine the solution from this table?</td>
<td>How can you confirm that the solution you identified is correct? Try to use equations to verify that your solution is correct.</td></tr>
<tr><td>Plots points and creates graphs.

</td>
<td>Tell me about your graph. What does the point of intersection represent in this problem situation?</td>
<td>Can you determine the exact solution from your graph? Try to use equations to verify that the point of intersection is the correct solution.</td></tr>
<tr><td>Writes the equations and uses guess-and-check.

Serena: $y = -90 + 1x$
Dion: $y = -1x$</td>
<td>Tell me about how you are using your equations to determine a solution.</td>
<td>What can you tell me about the y terms in your equations? Use that fact and your equations to find an algebraic method to efficiently determine where Serena and Dion will meet.</td></tr>
<tr><td>Writes the equations and solves algebraically using the properties of equality.

$-90 + 1x = -1x$
$1x = -1x + 90$
$2x = 90$
$x = 45$</td>
<td>Explain your solution strategy to me. Why did you set $-90 + 1x$ equal to $-1x$?

Why are the expressions still equal after each step?</td>
<td>Is $x = 45$ the solution to the system of linear equations that you wrote to model the situation in Part 1? Write an explanation about what is the answer to the problem in Part 1 and what is the solution to the system of linear equations that you wrote to model the situation in Part 1.</td></tr>
</table>

SHARE, DISCUSS, AND ANALYZE PHASE

LESSON GUIDE 3

Part 1

EU: The solutions to a linear equation in two variables can be represented graphically by a line consisting of all of the points represented by the ordered pairs (*x, y*) that make the equation a true statement or satisfy the equation.

EU: The solution to a system of two linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because the intersection point(s) make(s) both of the equations true statements or satisfies both of the equations in the system simultaneously.

Plots points and creates graphs:

- How did you create your graph and how does your graph represent the problem situation?
- How many solutions are there to the equation that represents Serena's line? *(There are infinitely many, because there are infinitely many points on the line.)* What about Dion's?
- Who can explain what it means to be a solution to the equation that represents each line?
- I'm hearing that there are infinitely many points on each of these lines, and that when you substitute any of these points into the equation that models the line, the result is a true statement. Another way to say that is that all the points on the line satisfy the equation. **(Recapping)**
- What is unique about this point of intersection right here? *(That's the point that's on both lines; That's a solution to both equations; That point makes both equations true.)*
- How do they know this point of intersection is a solution to both equations? *(The point is on both lines. The x- and y-values are the same.)* So when you say that the *x*- and *y*-values are the same, do you mean both are 45? **(Challenging)** *(No. The x-values are both 45 and the y-values are both -45.)*
- So we are saying that we are looking for a point that makes both equations true, or satisfies both equations, simultaneously. **(Revoicing and Marking)** We can also see how confusing it can get when we do not use the language of mathematics in our explanations. Let's try that question again. What is unique about this point of intersection? Let's hear the answer from several of you. *(Makes both equations true; Satisfies both equations; Simultaneously; It's the solution to each of the equations at the same time.)*
- Who can explain what this point of intersection represents in this problem?
- We discussed that we can identify a point of intersection graphically. The *x*- and *y*-values of a point of intersection will satisfy, or make true, both linear equations simultaneously and therefore the point of intersection is a solution to the system of linear equations. **(Recapping)**

LESSON GUIDE 3

EU: The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs (x, y) that make both equations true statements or satisfy the equations simultaneously.

EU: The properties of equality, including the reflexive, symmetric, and transitive properties of equality, along with substitution form the basis of algebraic methods (substitution and the addition/elimination) for finding the solution to a system of equations, or recognition that the system has no or an infinite number of solutions.

Writes the equations and uses guess-and-check:

- How do the equations, Serena: $y = -90 + 1x$ and Dion: $y = -1x$, represent the situation?
- How can we use these equations to confirm that (45, -45) is a point of intersection of the lines that represent these equations? *(If you substitute 45 into both equations, you get -45 as your y-value.)*
- I'm hearing that you can use the equations by substituting in input values and identifying which input value produces the same output value for both linear equations. **(Marking)** Can someone explain what I just said using the context of Serena and Dion's scuba dive?

Writes the equations and solves algebraically using the properties of equality:

- Do we need to use guess-and-check? Let's hear from a group that used the equations to algebraically determine this solution by applying the properties of equality. Explain your strategy to us. *(We set the expressions equal to each other and then added 90 to both sides, added 1x to both sides, and then divided both sides by 2.)*
- Can you just set the expressions equal to each other like that? Why? **(Challenging)** *(Yes, since both expressions are equal to y, then they must equal each other.)*
- Who remembers the name of the property of equality that says since both expressions are equal to y, then they must equal each other? **(Revoicing)** *(The transitive property of equality.)* **(Marking)**
- Can someone show us on this group's graph why we can set the expressions equal? *(The lines represent the equations and the y-values are the same at the point of intersection of the lines.)* Graphically, we can justify setting the expressions equal because at the intersection point the y-values are equal. **(Revoicing and Marking)**
- What about contextually? Who can point to words in the context that suggest we can set the y-values equal to each other? *(…after how many seconds will Dion and Serena be at the same depth?)*
- I have now heard three important ways to justify why we can set $-90 + 1x$ equal to $-1x$.

    1. Both expressions are equal to y, so by the transitive property of equality the expressions are equal to each other.
    2. At the point of intersection, the y-values must be equal.
    3. The context asks when the depths or y-values are equal. **(Revoicing)**

    Let's add solving algebraically to our chart and also add these justifications. **(Marking)**
- Is this the only way to solve this equation algebraically? **(Challenging)**
- What is the purpose of solving this equation algebraically? *(To get x by itself so we know what x is; Now we know the value of x at the point of intersection.)*
- So I'm hearing that we can solve a system of linear equations algebraically when the equations are in $y = mx + b$ form by setting the expressions, the $mx + b$ expressions, equal to each other and applying the properties of equality to isolate x. **(Recapping)** This method is called the substitution method for solving a system of linear equations. **(Marking)** Let's label the method on our chart.

LESSON GUIDE 3

Part 2

- Some students used Part 2 to help them determine that they could set the expressions equal to each other in Part 1 and then use the substitution method we just discussed. Please work in your groups on Part 2. When we come back together to discuss Part 2, please be prepared to describe each step and explain why each step makes sense algebraically.
- For Step 1, can someone remind us of the property of equality that states that we can set these expressions equal to one another? How else can we justify setting the expressions equal to one another?
- Let's continue with Step 2. *(In Step 2, 1.5x is added to both sides to get an equation with an x term on only one side of the equation. You can do this because the same amount is added to both sides.)* Why do we want an equation with an *x* term on only one side of the equation? **(Challenging)** *(We want to find out what 1x equals, so we need to have x on only one side of the equation.)*
- What happens in Step 3? *(In Step 3, 70 is added to both sides of the equation. This is OK too, because the equation is still balanced since the same amount of 70 was added to both sides.)* How does adding 70 to both sides help us? **(Challenging)** *(Now, the x term is all by itself on one side of the equation, so all we need to do is Step 4 and we can find out the value of 1x.)*
- Please say more about what's going on in Step 4. *(In Step 4, the terms on both sides of the equal sign are divided by 3.5 so that only 1x is left on the left side of the equation. The equation is still balanced because we divided both sides of the equation by the same amount.)*
- Now in Step 5 we have $x = 20$. What does this mean in the context of the problem? *(20 seconds is the time when Serena and Dion are at the same depth.)* How did the substitution method allow us to determine the *x* value of the point of intersection? *(Once we set Serena's and Dion's expressions equal to one another to make an equation, we can solve the equation to find the value of x.)*
- Is $x = 20$ the solution to the system of linear equation? *[No, it's just the x-value of the solution. The solution is (20, -30).]* Explain how you determined that (20, -30) is the solution. *(Since 20 is the value of x when the Serena and Dion are at the same depth, we evaluated each of their equations at x = 20 and found that both equal a depth of -30.)*
- I'm again hearing that we can solve a system of linear equations algebraically when the equations are in $y = mx + b$ form by setting the expressions, the $mx + b$ expressions, equal to each other and applying the properties of equality to isolate *x*. We can then evaluate the equations at this value of *x* in order to determine the value of *y* at the point of intersection, and that a solution to the system of linear equations is this ordered pair (x, y) that makes both equations true simultaneously. **(Recapping)**

LESSON GUIDE 3

Application	Suppose on a different day, Serena's dive is represented by the equation $y = -50 + 1.5x$ and Dion's dive is represented by the equation $y = -1x$, where y represents depth in feet and x represents time in seconds. After how many seconds will the divers be at the same depth? Show all work and explain your reasoning.
Summary	How can you use the substitution method to determine the solution to a system of linear equations?
Quick Write	Recall from the Task 3 Application: *Suppose on a different day, Serena's dive is represented by the equation $y = -50 + 1.5x$ and Dion's dive is represented by the equation $y = -1x$, where y represents depth in feet and x represents time in seconds.* Describe what each term of each equation means in the context of Serena's and Dion's dives.

English Learner Support:

1. Display images or video of scuba divers so that students associate the words in the problem with the images.
2. Continue to add to the English and appropriate home language math vocabulary list. For this task, include "satisfy an equation," "simultaneously," "substitution," and "transitive property of equality".
3. Discuss multiple representations and use teacher prompts to connect the multiple representations to help all students make sense of the mathematical relationships.
4. Organize the classroom display so that students can easily compare solution methods.
5. Slow the pace of the discussion and hold all students accountable for listening and saying back the ideas of others. Ask students to repeat ideas, to put ideas in their own words, and point to graphs, tables, equations, and, for Part 2, explanations for why each step makes sense algebraically.
6. Continue to refer and add to the poster titled, "Strategies for Solving Systems of Linear Equations". Include advantages and disadvantages of the strategy as those ideas emerge in the discussion, and ask students who are English learners to step back and notice similarities, differences, and/or patterns.