Using Talk to Make Sense of Mathematics

By Victoria Bill and Laurie Speranzo

When students talk about their solution paths and other’s solution paths they learn the most.

The students below in the classroom vignette are learning a great deal because they are talking about models and making sense of mathematics. The teacher is also learning about student thinking and reasoning. See what you understand about student understanding for division of a fraction by a fraction.

Students are explaining that \( \frac{1}{2} \div \frac{1}{4} \) is 2. A student says,

S1: It is 2 because if you think about a half of a cake you want to figure out how many fourths are in the half cake and there are two of them.

T: Who understood and can put the idea into their own words?

S2: She said there is half and inside half if you cut it into two pieces there are two smaller pieces.

S3: Those two pieces are fourths.

T: Can anyone add on to what she said?

S4: Can I make a drawing?

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T: Can anyone add on to what she said? You can do 1/4 x 4 and you get 2 of them.

S5: I don't understand why you did that.

T: Who can explain what she did?

S2: He knows that fourths mean there will be four pieces so he is figuring out how many fourths are in the whole and the half.

S4: There are four pieces in the whole but we only have half so this means now there are two fourths in the half.

T: I need to hear that back a few more times to make sure I understand the thinking.
We know it sounds trivial, but more and more we are convinced that when a student shares their work that all students benefit when there are follow-up opportunities for between 5 to 8 students to:

- say back what they heard,
- add on,
- make a drawing,
- say it in their own words,
- agree or disagree, and
- summarize the ideas.

Had the teacher moved forward and just accepted the first student’s response, many other students in the classroom might not have understood or been totally clear about why the quotient to \(\frac{1}{2} \div \frac{1}{4}\) was 2. The teacher would also have missed an opportunity to learn about student thinking.

The Institute for Learning calls the process of hearing from many students in the classroom, talk that is “Accountable to Community.”

You may wonder why the teacher doesn’t just have each group share their solution path for solving the task and then talk after all of the solution paths have been shared. After all, the students did all work on the same task, so they do have some knowledge of what is being discussed; they just may not understand the approach or the model used by another group. Exactly! The other groups took different approaches and used different models that students in the class can benefit from learning. The solution path shared above was not one that belonged to all of the students in the classroom; it only belonged to the group sharing the solution path. If students did not use this pathway, they need to hear it, probably more than once, in order to understand this pathway that they did not use themselves. By understanding others’ solution paths they can make comparisons to their own pathway. And by making connections between their solution paths and other students’ solution paths, the chances of the students deepening their understanding of the concept increases and students learn additional strategies and models for solving problems.

Imagine the teacher has a student who created a number line share next.
This visual model looks incredibly different from the area model introduced first. A different representation of the whole is shown with two one-fourths contained within the one-half. How many students would need to point to and explain what is happening with the number line model before the teacher can be sure that all of the students have a true understanding of the math in the model? Probably at least 5 if not 8 out of a class of 20.

Only after discussing both models can the teacher finally ask students to compare and contrast the models. If you do not understand each solution path, you cannot make comparisons between solution paths. By asking students to say more, to add on, to agree or disagree with their peer’s solution paths the teacher can focus on the uptake of student ideas (cf. Applebee, Langer, Nystrand, & Gamoran, 2003; Nystrand, 2006; Soter et al., 2008).

Engle and Conant (2002) show that productive disciplinary engagement occurs in learning environments characterized by: (1) giving students authority to address problems, (2) holding students accountable to others and to shared disciplinary norms (3) problematizing subject matter by asking students to compare and contrast solution paths, and (4) providing students with relevant resources such as context, manipulatives or the freedom to make diagrams or number lines.

Who benefits when 5-8 people say back the strategy/solution path? Students get to hear ideas said more than one way. When ideas are said in less than clear ways, students can count on their peers to add on to the line of reasoning and together students co-construct sound mathematical explanations which they might not have been able to do independently. For some students, giving the opportunity to hear from several students acts as wit time for students to process their own thoughts. The students who need to hear things different ways get that exposure. (And weren’t we all that person in some class or another, thinking to ourselves “What did they just say? I’m not following.”?)
So, what’s the down-side? What about the student who “got it” the first time? They get to have time to process and add to their own thinking or see things that others do that they did not even occur to them as THEY were doing it! There is no down-side! Give it a try!

Remember these five key questions:

- Can you say back what was just said?
- Can you put those ideas in your own words?
- What did you understand in the explanation we just heard?
- Who can add on to what was just said?
- Do you agree or disagree and say why?

Your Turn

Engage students in solving and discussing the solution paths to a high-level mathematic task. If the task is a high-level task, there will be multiple ways students can solve the task. Remember, when you have a solution path shared, you must call on 5 to 8 students by using the questions above. This means you may use a question more than once. Write and tell all of us what happens in your classroom as several students say and say again in their own words what they heard. Please share in the comments section below or reach out to Victoria Bill or Laurie Speranzo.